TRANSVERSE MOMENTUM DEPENDENCE OF CUMULATIVE PIONS *

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Abstract

In the framework of the recently proposed QCD based parton model for the cumulative phenomena in the interactions with nuclei the dependence of the cumulative pion production rates on the transverse momentum $K_\perp$ is studied. The mean value of $K_\perp$ is found to grow with x in the cumulative region. The obtained results are in agreement with the available experimental data.

*This work is supported by the Russian Foundation for Fundamental Research, Grant No. 97-02-18123

1 Introduction

A few years ago we have proposed a quark-parton model of cumulative phenomena in the interactions with nuclei [1, 2] based on perturbative QCD calculations of the corresponding quark diagrams near the thresholds, at which other quarks ("donors") in the nuclear flucton transfer all their longitudinal momenta to the distinguished active quark and become soft.

Consider the scattering of a hadronic projectile off a nucleus with the momentum P in the c.m. system. At high energies the momentum K of the produced pion belongs to the cumulative region if $K_z > P_z/A$. As a reasonable first approximation, we treat the nucleus as a collection of $N = 3A$ valence quarks, which, on the average, carry each longitudinal momentum $x_0 P_z/A$ with $x_0 = 1/3$. In our approach the cumulative pion production proceeds in two steps. First a valence quark with a scaling variable $x > 1$ is created. Afterwards it decays into the observed hadron with its scaling variable x close to the initial cumulative quark's one. This second step is described by the well-known quark fragmentation functions [3] and will not be discussed here.

The produced cumulative ("active") quark acquires the momentum much greater than $x_0 P_z/A$ only if this quark has interacted by means of gluon exchanges with other p quarks of flucton ("donors") and has taken some of their longitudinal momenta (see Fig.1). If this active quark accumulates all longitudinal momentum of these p quarks then $K_z = (p+1)x_0 P_z/A$ and the donors become soft. It is well-known that interactions which make the longitudinal momentum of one of the quark equal to zero may be treated by perturbation theory [4]. This allows to calculate the part of Fig. 1 responsible for the creation of a cumulative quark explicitly. This was done in [1, 2], to which papers we refer the reader for all the details. As a result we were able to explain the exponential fall-off of the production rate in the cumulative region.

Since with the rise of x the active quark has to interact with a greater number of donors, one expects that its average transverse momentum also grows with x. Roughly one expects that $\langle K_\perp^2 \rangle$ is proportional to the number of interactions, that is, to x. In

[1, 2] this point was not studied: we have limited ourselves with the inclusive cross-section integrated over the transverse momenta, which lead to some simplifications. The aim of the present paper is to find the pion production rate dependence on the transverse momentum and the mean value of the latter as a function of x in the cumulative region. This dependence and also the magnitude of $\langle K_\perp^2 \rangle$ have been studied experimentally. The comparison of our predictions with the data allows to obtain further support for our model and fix one of the two its parameters (the infrared cutoff).

2 The $K_\perp$ dependence

Repeating the calculations of the diagram in Fig.1 described in [1, 2] but not limiting ourselves with the inclusive cross-section integrated over the transverse momentum, we readily find that all dependence upon the transverse momentum $K_\perp$ of the produced particle is concentrated in a factor:

$$J(K_\perp) = \int \rho_A(\underbrace{r, ..., r}_{p+1} | \underbrace{\overline{r}, ..., \overline{r}}_{p+1}) G(c_1, ..., c_p) \prod_{i=1}^{p} \lambda(c_i - r)\lambda(c_i - \overline{r}) d^2c_i e^{i(\overline{r}-r)K_\perp} d^2r d^2\overline{r} \quad (1)$$

Here ρ_A is the (translationally invariant) quark density matrix of the nucleus:

$$\rho_A(r_i|\overline{r}_i) \equiv \int \psi_{\perp A}(r_i, r_m)\psi^*_{\perp A}(\overline{r}_i, r_m) \prod_{m=p+2}^{N} d^2r_m \quad (2)$$

where $\psi_{\perp A}$ is the transverse part of the nuclear quark wave function. The propgation of soft donor quarks is decribed by

$$\lambda(c) = \frac{K_0(m|c|)}{2\pi} \quad (3)$$

where m is the constituent quark mass and K_0 is the modified Bessel function (the MacDonald function). The interaction with the projectile contributes a factor

$$G(c_1, ..., c_p) = \int \prod_{i=1}^{p} \sigma_{qq}(c_i - b_i)\eta_H(b_1, ..., b_p) d^2b_i \quad (4)$$

where $\sigma_{qq}(c)$ is the quark-quark cross-section at a given value of impact parameter c and

$$\eta_H(b_1, ..., b_p) = \sum_{L\geq p} \frac{L!}{(L-p)!} \int |\psi_{\perp H}(b_i)|^2 \delta^{(2)}(\frac{1}{L}\sum_{i=1}^{L} b_i)\ d^2b_{p+1}...d^2b_L \quad (5)$$

is a multiparton distribution in the projectile, expressed via the transverse part of its partonic wave function $\psi_{\perp H}$. If we integrate $J(K_\perp)$ over $K_\perp$ we come back to our old result (Eq. (33) in [1]):

$$\int J(K_\perp)\frac{d^2K_\perp}{(2\pi)^2} = \rho_A(\underbrace{0,...,0}_{p+1}|\underbrace{0,...,0}_{p+1})\int G(c_1,...,c_p)\prod_{i=1}^{p}\lambda^2(c_i-r)d^2c_id^2r$$

If one assumes factorization of the multiparton distribution $\eta_H(b_1,...,b_p)$ then $G(c_1,...,c_p)$ also factorizes:

$$G(c_1,...,c_p) = \prod_{i=1}^{p} G_0(c_i) \tag{6}$$

Following [2] we use the quasi-eikonal approximation for η_H:

$$\eta_H(b_1,...,b_p) = \xi^{(p-1)/2}\nu_H^p\prod_{i=1}^{p}\eta_H(b_i)$$

where ξ is the quasi-eikonal diffraction factor, ν_H is the mean number of partons in the projectile hadron and the single parton distribution $\eta_H(b)$ is normalized to unity. In a Gaussian approximation for $\sigma(c)$ and $\eta_H(b)$ we find:

$$G_0(c) = \xi^{\frac{1}{2}-\frac{1}{2p}}\frac{\nu_H\sigma_{qq}}{\pi r_{0H}^2}e^{-\frac{c^2}{r_{0H}^2}}$$

where σ_{qq} is the total quark-quark cross-section, $r_{0H}^2 = r_0^2 + r_H^2$, r_0 and r_H are the widths of $\sigma(c)$ and $\eta_H(b)$ respectively.

With the factorised $G(c_1,...,c_p)$ (6) we have

$$J(K_\perp) = \int \rho_A(\underbrace{0,...,0}_{p+1}|\underbrace{\overline{r}-r,...,\overline{r}-r}_{p+1})j^p(r,\overline{r})e^{i(\overline{r}-r)K_\perp}d^2rd^2\overline{r}$$

where

$$j(r,\overline{r}) = \int d^2cG_0(c)\lambda(c-r)\lambda(c-\overline{r})$$

We also have used the translational invariance of the ρ-matrix. Note that near the real threshold we have no spectators and

$$\rho_A(\underbrace{0,...,0}_{p+1}|\underbrace{\overline{r}-r,...,\overline{r}-r}_{p+1}) = \rho_A(\underbrace{0,...,0}_{p+1}|\underbrace{0,...,0}_{p+1})$$

In any case large $K_\perp$ corresponds to small $\overline{r}-r$ so we factor ρ_A out of the integral in zero point. In the rest integral we pass to the variables

$$B = \frac{r+\overline{r}}{2}, \qquad b = \overline{r} - r$$

and shift the integration variable c, then

$$J(K_\perp) = \rho_A(\underbrace{0,...,0}_{p+1}|\underbrace{0,...,0}_{p+1}) \int j^p(B,b)e^{ibK_\perp}d^2bd^2B \tag{7}$$

where

$$j(B,b) = \int G_0(B+c)\lambda(\frac{b}{2}-c)\lambda(\frac{b}{2}+c)d^2c \tag{8}$$

3 The calculation of $\langle |K_\perp| \rangle$

Now we would like to find the width of the distribution on $K_\perp$ as a function of p or what is the same of the cumulative number $x = (p+1)/3$. From the mathematical point of view it is simpler to calculate the mean squared width of the distribution $\langle K_\perp^2 \rangle$. Unfortunately in our case this quantity is logarithmically divergent at large $K_\perp$. This divergency results from the behavior of $j(B,b)$ at small b. This behavior is determined by the behavior of the $\lambda(b) = K_0(m|b|)/(2\pi)$ (3), which has a logarithmical singularity at $|b| = 0$. Smooth $G_0(B+c)$ does not affect this behavior.

For this reason we shall rather calculate $\langle |K_\perp| \rangle$:

$$\langle |K_\perp| \rangle = \frac{1}{J_N}\int j^p(B,b)|K_\perp|e^{ibK_\perp}d^2bd^2B\frac{d^2K_\perp}{(2\pi)^2} \tag{9}$$

where J_N is the same integral as in the numerator but without $|K_\perp|$. Presenting $|K_\perp|$ as $K_\perp^2/|K_\perp|$ and $K_\perp^2$ as the Laplacian Δ_b applied to the exponent we find

$$\langle |K_\perp| \rangle = -\frac{1}{J_N}\int j^p(B,b)\Delta_b e^{ibK_\perp}d^2bd^2B\frac{d^2K_\perp}{|K_\perp|(2\pi)^2}$$

Twice integrating by parts and using the formula

$$\int \frac{d^2K_\perp}{|K_\perp|}e^{ibK_\perp} = \frac{2\pi}{|b|}$$

we find

$$\langle|K_\perp|\rangle = -\frac{1}{2\pi J_N}\int \frac{1}{|b|}\Delta_b j^p(B,b)d^2b d^2B$$

Now we again integrate by parts once to find

$$\langle|K_\perp|\rangle = -\frac{1}{2\pi J_N}\int d^2B\frac{d^2b}{|b|^2}(n_b\nabla_b)j^p(B,b)$$

where $n_b = b/|b|$. This leads to our final formula

$$\langle|K_\perp|\rangle = -\frac{p}{2\pi J_N}\int d^2B\frac{d^2b}{|b|^2}j^{p-1}(B,b)(n_b\nabla_b)j(B,b) \tag{10}$$

where $j(B,b)$ is given by (8), $\lambda(b)$ is given by (3) and

$$J_N = \int d^2Bj^p(B,b=0)$$

4 Approximations

To simplify numerical calculations we make some additional approximations, which are not very essential but are well supported by the comparison with exact calulations at a few sample points.

As follows from the the asymptotics of $K_0(z)$ at large z

$$K_0(z) \simeq \sqrt{\frac{\pi}{2z}}e^{-z}$$

the width of $\lambda(b)$ (3) is of the order m^{-1}. The function G_0 is smooth in the vicinity of the origin and its width $r_{0H} = \sqrt{r_0^2 + r_H^2}$ is substancially larger than the width of λ. For this reason we factor $G_0(B+c)$ out of the integral (8) over c at the point B:

$$j(B,b) = G_0(B)\Lambda(b), \qquad \Lambda(b) \equiv \int \lambda(c)\lambda(c-b)d^2c = \frac{|b|}{4\pi m}K_1(m|b|) \tag{11}$$

Then we find that the integrals over B and b decouple

$$J(K_\perp) = \rho_A(\underbrace{0,...,0}_{p+1}|\underbrace{0,...,0}_{p+1})\int G_0^p(B)d^2B\int \Lambda^p(b)e^{ibK_\perp}d^2b \tag{12}$$

In this approximation we find that $\langle|K_\perp|\rangle$ depends only on one parameter - the constituent quark mass m, which in our approach plays the role of an infrared cutoff:

$$\langle|K_\perp|\rangle = pm\int_0^\infty dzK_0(z)(zK_1(z))^{p-1} \tag{13}$$

This allows to relate m directly to the experimental data on the transverse momentum dependence.

5 Comparison with the data and discussion

The integral in (13) can be easy calculated numerically. For values of $p = 1, ..., 12$ it is very well approximated by a power dependence (see Fig.2), so that we obtain

$$\langle|K_\perp|\rangle/m = 1.594\, p^{0.625} \tag{14}$$

As we observe, the rise of $\langle|K_\perp|\rangle$ turns out to be even faster than expected on naive physical grounds mentioned in the Introduction ($\sim \sqrt{p}$). The resulting plots for $\langle|K_\perp|\rangle^2$ as a function of the cumulative number $x = (p+1)/3$ at different values of parameter m are shown in Fig.3 together with avaiable experimental data from [5] on $\langle K_\perp^2\rangle$ for pion production obtained in experiments [5]-[7] with 10 GeV protons and [8, 9] with 8.94 GeV protons.

Note that earlier publications of the first group [6, 7] reported a much stronger increase of $\langle K_\perp^2\rangle$ with x, up to value 2 $(GeV/c)^2$ at $x = 3$ for pion production. In our approach such an increase would require the quark mass to be as high as $m \simeq 225MeV$. In a more recent publication [5] the rise of $\langle K_\perp^2\rangle$ is substancially weaker (it corresponds to $m \simeq 175MeV$ in our approach). The authors of [5] explain this by new experimental data obtained and by a cutoff $K_{\perp max}$ introduced in calculations of $\langle K_\perp^2\rangle$ in [5]. The introduction of this cutoff considerably (approximatly two times) decreases the experimental value of $\langle K_\perp^2\rangle$ at $x = 3$. In our opinion this is a confirmation that the cumulative pion production rate only weakly decreases with $K_\perp$ in the cumulative region so that the the integral over $K_\perp^2$ which enters the definition of $\langle K_\perp^2\rangle$ is weakly convergent or even divergent, as in our approach. Undoubtedly presentation of the experimental data in terms of the mean value $\langle|K_\perp|\rangle^2$, rather than $\langle K_\perp^2\rangle$ should reduce the dependence on the cutoff $K_{\perp max}$ and make the results more informative.

One of the ideas behind the investigations of the cumulative phenomena is that they

may be a manifestation of a cold quark-gluon plasma formed when several nucleons overlap in the nuclear matter. In [1] we pointed out that our model does not correspond to this picture. It implies coherent interactions of the active quark with donors and, as a result, strong correlations between the longitudinal and transverse motion. Predictions for the dependence of $\langle|K_\perp|\rangle$ on x are also different. From the cold quark-gluon plasma model one expects $\langle|K_\perp|\rangle$ to behave as $x^{1/3}$, since the Fermi momentum of the quarks inside the overlap volume is proportional to the cubic root of the quark density. Our model predicts a much faster increase, with a power twice larger. The experimental data seem to support our predictions.

6 Acknowledgments

The authors are greatly thankful to Prof. P.Hoyer who attracted their attention to the problem.

This work is supported by the Russian Foundation for Fundamental Research, Grant No. 97-02-18123.

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Figure captions

Fig. 1 The diagram for the production of a cumulative quark with the momentum K in the scattering of a projectile hadron with the momentum H off a nucleus A with the momentum P. Dashed and chain lines show gluon and pomeron exchanges, respectively.

Fig. 2 The $\langle|K_\perp|\rangle/m$ as a function of p. The points are the results of calculations on (13). The line is the best power fit.

Fig. 3 The $\langle|K_\perp|\rangle^2$ as a function of the cumulative number $x = (p+1)/3$. The lines are the results of calculations on (13) at different values of parameter m. The points ($\bullet$) are the experimental data from [5] on $\langle K_\perp^2\rangle$ for pion production with a cutoff (see the text) obtained in experiments [5]-[9] on a bombardment of nuclei by 10 GeV and 9 GeV protons. The points ($\circ$) are the data from the earlier publications of the group [6, 7] without a cutoff.

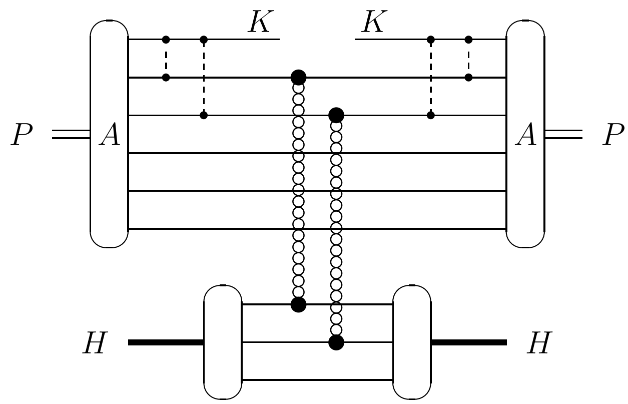

Fig. 1

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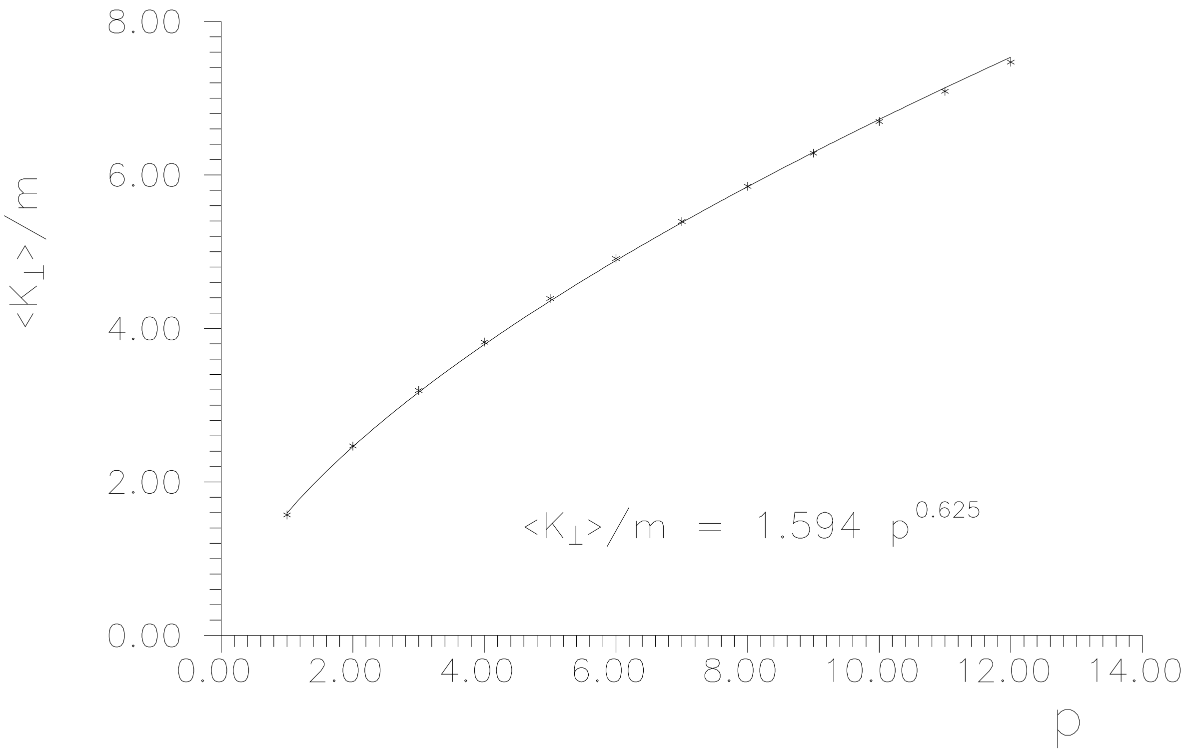

Fig. 2

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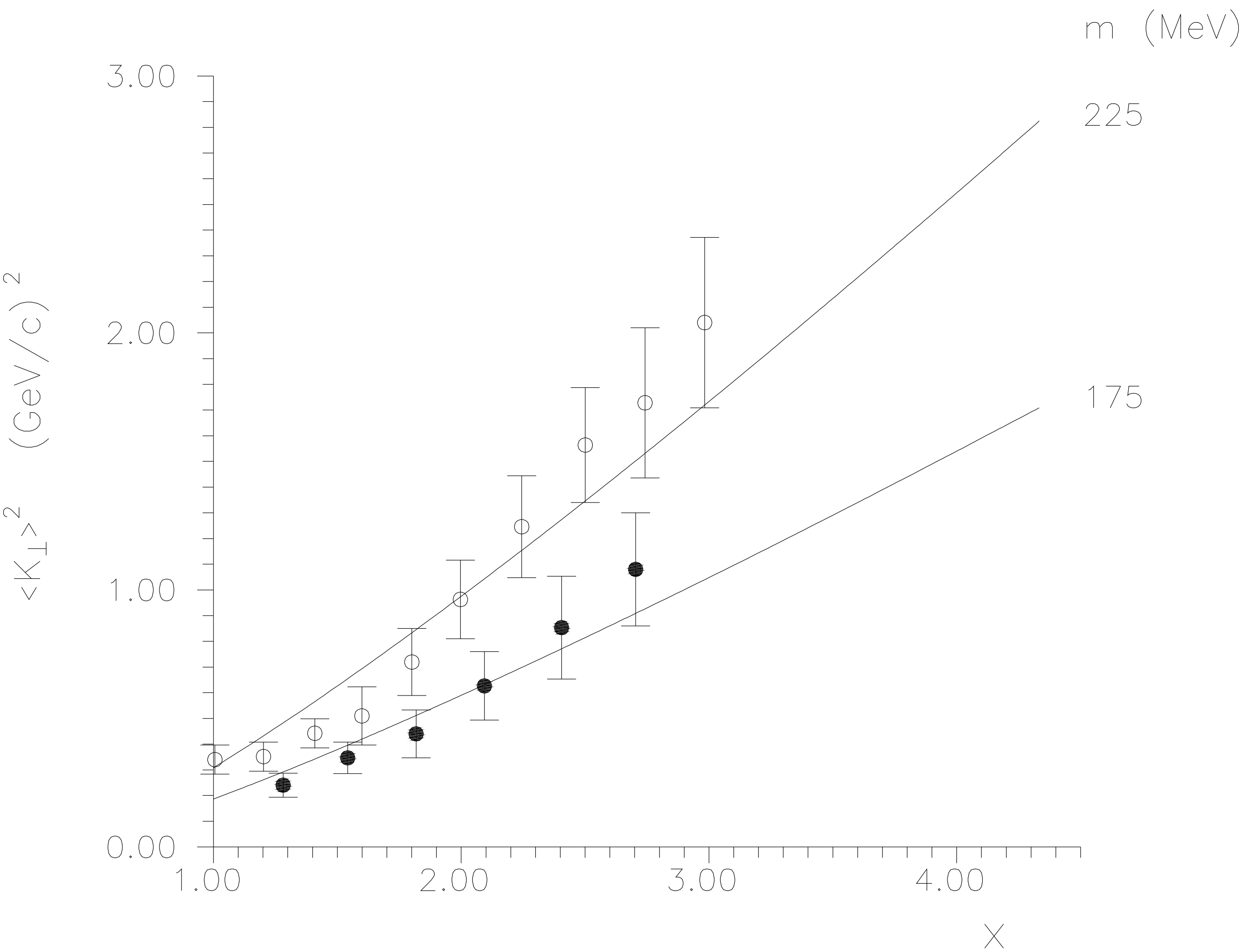

Fig. 3

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